Assignment #1 – Due Thursday March 18

1. Prove that if the short-interest-rate is constant, the forward price and the futures price are identical.
   - Recall the difference between the two contracts is the timing of cash-flows. A forward contract has all cash-flows at date $T$. A futures contract has a settlement and re-contract each day (or instant if you prefer).
   - It might help to consider a simple two period contract first.
   - More generally, what are the necessary and sufficient conditions on the interest rate that imply forward and futures prices are identical.
   - You should think about this for a bit. At some point you may wish to consult the Cox, Ingersol, Ross (1981). It is [12] on the syllabus.

2. Read some of the papers in Empirical Facts about Commodities. (For Example, take a look at [6], [7] and [10]). Briefly summarize the stylized facts regarding commodities. (The key here is brief!! Point form is sufficient).

3. It is often stated that commodity prices are not that sensitive to interest rates. Compare the variance of the short-rate to the variance of a typical commodity price series. Use some of the results in [1] or [2] to construct a “variance bound” by decomposing the pricing relationship.
   - This is the reason that forward-futures distinction discussed in question 1 is not that important.

4. In case you were wondering, yes you can buy derivatives on the weather (see attached add from a recent WSJ). This question deals with the pricing of these contracts and the completeness of the market with respect to weather. In particular, we will focus on temperature forwards. Here is the structure of the economy

   Time is discrete and $t=1,\ldots,T$. The temperature at any date $t$ is $a_t \in O$. We make the simplifying assumption that $O$ is a finite set. In addition we assume that the
temperature follows a Markov process with transition matrix $\Pi$. The elements of $\Pi$ are $\pi(a_i | a_j)$ representing the probability of moving to a temperature of $a_i$ next period given the current temperature is $a_j$. Let the risk-free interest rate be zero. Assume risk-neutral traders.

Show:
(a) $F_{t,T} = E[a_T | a_t]$
(b) Consider the case where $O=\{50,75\}$. Pick some sensible values for $\Pi$ and plot the forward curves (forward prices at different horizons). What are all the shapes you can generate.
(c) Show $\text{Var}(F_{t+1,T+1} | a_t) \leq \text{Var}(F_{t+1,T} | a_t)$. Show that the conditional variance of the forward prices decreases with maturity (when is the inequality strict). What is the intuition for this result? (Hint: What is $F_{t+1,\infty}$?)
(d) Show that using any arbitrary set of just forward contracts (i.e., the one period, the two period etc.) CANNOT span or complete the market.
\quad ° For dynamic spanning see Huang and Litzenberger page 197 ff.
\quad ° See Kreps (1982) in McCall The economics of uncertainty.
(e) Consider the case where $O=\{50,75\}$. Show that the market is dynamically complete using a forward contract of any maturity and a risk-free bond. What is needed in order to be able to complete the market using any forward contract and a bond.
(f) Under what conditions might the risk-neutral assumption be reasonable? Do you think it is reasonable for weather derivatives? Briefly explain how you might test this assumption.

Think about – but you do not need to hand-in
\quad ° In an economy with a finite horizon $T$, where $O$ has $m$ states, is it the case that a risk-free bond and $m-1$ forward contracts can dynamically complete market generically (where generically means that if we selected $O$ and $\Pi$ randomly, the market is incomplete only on a measure zero set of economies)
\quad ° see Kreps (1982)).

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**Enron, ENW Of The Netherlands Sign**
Weather Risk Mgmt Pact
March 3, 1999
Dow Jones Newswires

LONDON -- Enron Europe, a subsidiary of Enron Corp., signed its first weather risk management contract with Energie Noord West NV (ENW) of the Netherlands, it said in a news release Wednesday.

The contract hedges financial risk against unpredictable weather taking into consideration temperature, rainfall and other climactic changes.

Natural gas and electricity prices are strongly co-related to the changes in weather and ENW is one of the largest suppliers of both commodities in the Netherlands.

Enron is an integrated energy company and has concluded 350 such weather derivatives contracts in the U.S.